

# Supersymmetric Axion-Neutrino Merger

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## Abstract

The recently proposed supersymmetric  $A_4$  model of the neutrino mass matrix is modified to merge with a previously proposed axionic solution of the strong CP problem. The resulting model has only one input scale, i.e. that of  $A_4$  symmetry breaking, which determines both the seesaw neutrino mass scale and the axion decay constant. It also solves the  $\mu$  problem and conserves  $R$  parity automatically.

# 1 Introduction

There are a number of issues in particle physics which require theoretical clarification to go along with our present experimental knowledge. As it stands, the minimal standard model (SM) has the following shortcomings: (1) absence of neutrino mass, (2) presence of the strong CP problem, and (3) presence of the hierarchy problem. Supersymmetry is widely assumed to be the resolution of (3), but the minimal supersymmetric standard model (MSSM) still has no cure for (1) or (2). Furthermore, it raises two other issues not present in the SM, namely (4) the nonconservation of baryon number and lepton number without the imposition of  $R$  parity, and (5) the  $\mu$  problem, i.e. why the allowed supersymmetric term  $\mu\hat{\phi}_1\hat{\phi}_2$  in the superpotential has  $\mu$  of the order of the supersymmetry breaking scale, instead of some other much larger scale such as the Planck scale or the string scale.

In this paper, two recent proposals [1, 2] are merged to form a cohesive theoretical understanding of all 5 issues. The resulting supersymmetric model has only one input mass scale in the superpotential, which determines the anchor mass of the neutrino seesaw mechanism [3] as well as the scale of the spontaneous breaking of an anomalous global  $U(1)$  symmetry [4]. The latter solves the strong CP problem [5] and predicts the existence of a very light pseudoscalar particle, the axion [6]. Its specific implementation [1] in this model also solves the  $\mu$  problem and leads to the automatic conservation of  $R$  parity.

## 2 Description of Model

The standard  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge symmetry is extended to include supersymmetry as well as an anomalous global  $U(1)_{PQ}$  symmetry and the discrete non-Abelian symmetry  $A_4$  [7], which has 4 irreducible representations, i.e.  $\underline{1}$ ,  $\underline{1}'$ ,  $\underline{1}''$ , and  $\underline{3}$ . The usual

superfields of the MSSM have the following assignments under  $U(1)_{PQ}$  and  $A_4$ :

$$\hat{Q}_i = (\hat{u}_i, \hat{d}_i), \quad \hat{L}_i = (\hat{\nu}_i, \hat{e}_i) \sim (1/2, \underline{\mathbf{3}}), \quad \hat{\phi}_{1,2} \sim (-1, \underline{\mathbf{1}}), \quad (1)$$

$$\hat{u}_1^c, \hat{d}_1^c, \hat{e}_1^c \sim (-1/2, \underline{\mathbf{1}}), \quad \hat{u}_2^c, \hat{d}_2^c, \hat{e}_2^c \sim (-1/2, \underline{\mathbf{1}}'), \quad \hat{u}_3^c, \hat{d}_3^c, \hat{e}_3^c \sim (-1/2, \underline{\mathbf{1}}''). \quad (2)$$

The following quark, lepton, and Higgs superfields are then added:

$$\hat{U}_i, \hat{U}_i^c, \hat{D}_i, \hat{D}_i^c, \hat{E}_i, \hat{E}_i^c, \hat{N}_i^c \sim (1/2, \underline{\mathbf{3}}), \quad (3)$$

$$\hat{\chi}_i \sim (0, \underline{\mathbf{3}}), \quad \hat{\zeta}_i \sim (-2, \underline{\mathbf{3}}), \quad \hat{S}_1 \sim (-1, \underline{\mathbf{1}}), \quad \hat{S}_2 \sim (2, \underline{\mathbf{1}}), \quad (4)$$

which are all  $SU(2)_L$  singlets. The superpotential of this model is then given by

$$\begin{aligned} \hat{W} = & \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3 + f_0 \hat{S}_2 \hat{\zeta}_i \hat{\chi}_i + f_1 \hat{S}_1 \hat{S}_1 \hat{S}_2 \\ & + f_2 \hat{S}_2 \hat{\phi}_1 \hat{\phi}_2 + \frac{1}{2} f_N \hat{S}_1 \hat{N}_i^c \hat{N}_i^c + f_\nu \hat{L}_i \hat{N}_i^c \hat{\phi}_2 \\ & + f_E \hat{S}_1 \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{N}_i^c \hat{\phi}_1 + h_{ijk}^e \hat{E}_i \hat{e}_j^c \hat{\chi}_k \\ & + f_U \hat{S}_1 \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h_{ijk}^u \hat{U}_i \hat{u}_j^c \hat{\chi}_k \\ & + f_D \hat{S}_1 \hat{D}_i \hat{D}_i^c + f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_1 + h_{ijk}^d \hat{D}_i \hat{d}_j^c \hat{\chi}_k. \end{aligned} \quad (5)$$

Note that the only allowed mass term is  $M_\chi$ . The heavy singlet quarks and leptons will have masses proportional to  $\langle S_1 \rangle$  and the  $\mu$  parameter will be proportional to  $\langle S_2 \rangle$ . Because of the choice of  $U(1)_{PQ}$ , the conservation of  $R$  parity is automatic as well.

### 3 Breaking of $A_4$ and $U(1)_{PQ}$

Consider the scalar potential consisting of the fields  $\chi_i$ ,  $\zeta_i$ ,  $S_1$ , and  $S_2$ .

$$\begin{aligned} V = & |M_\chi \chi_1 + h_\chi \chi_2 \chi_3 + f_0 S_2 \zeta_1|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1 + f_0 S_2 \zeta_2|^2 \\ & + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2 + f_0 S_2 \zeta_3|^2 + |f_0 S_2 \chi_1|^2 + |f_0 S_2 \chi_2|^2 + |f_0 S_2 \chi_3|^2 \\ & + |2f_1 S_1 S_2|^2 + |f_0 (\zeta_1 \chi_1 + \zeta_2 \chi_2 + \zeta_3 \chi_3) + f_1 S_1^2|^2. \end{aligned} \quad (6)$$

This has the supersymmetric solution ( $V = 0$ ) for

$$u_0 = \langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = -M_\chi/h_\chi, \quad (7)$$

and

$$v_2 = 0, \quad f_0 u_0 (u_1 + u_2 + u_3) + f_1 v_1^2 = 0, \quad (8)$$

where  $v_{1,2} = \langle S_{1,2} \rangle$  and  $u_i = \langle \zeta_i \rangle$ .

As shown in Ref. [1], in the presence of soft supersymmetry breaking, Eq. (8) will be modified to read

$$v_2 \sim M_{SUSY}, \quad f_0 u_0 (u_1 + u_2 + u_3) + f_1 v_1^2 \sim M_{SUSY}^2, \quad (9)$$

with  $v_1$  and  $u_i$  of order  $u_0$ . Going back to Eq. (5), this means that the heavy singlet quarks and leptons will all have masses of order  $M_\chi$ , but the  $\mu$  parameter is of order  $M_{SUSY}$ .

As shown in Ref. [2], the quarks and charged leptons will get Dirac seesaw masses from their heavy counterparts, and because of the  $A_4$  symmetry and Eq. (7), they are diagonalized by the same unitary matrix, i.e.

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (10)$$

where  $\omega = e^{2\pi i/3}$ . This means that the quark mixing matrix is just the identity matrix at this high scale. It also means that the neutrino mass matrix is given by

$$\mathcal{M}_\nu = \frac{f_\nu^2 \langle \phi_2 \rangle^2}{f_N v_1} U_L^T U_L = \frac{f_\nu^2 \langle \phi_2 \rangle^2}{f_N v_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (11)$$

As shown in Ref. [2], the one-loop radiative corrections of this matrix from the high scale to the electroweak scale automatically produce the phenomenologically favored neutrino mixing matrix with  $\theta_{atm} = \pi/4$  and  $\theta_{sol}$  large but less than  $\pi/4$ . In the model of Ref. [2], the heavy

singlet fermion masses are independent of one another and unrelated to the  $A_4$  breaking scale. Here, they are all determined to be of order  $M_\chi$ . Furthermore, the extra softly broken discrete  $Z_3$  symmetry used there is no longer needed. Both improvements are directly due to the specific way in which  $U(1)_{PQ}$  has been realized.

The axion of this model is contained mostly in the superfield

$$\frac{v_1 \hat{S}_1 + (2/3)(u_1 + u_2 + u_3)(\hat{\zeta}_1 + \hat{\zeta}_2 + \hat{\zeta}_3)}{\sqrt{v_1^2 + (4/3)(u_1 + u_2 + u_3)^2}}. \quad (12)$$

However, as  $S_2$  picks up a vacuum expectation value of order  $M_{SUSY}$  and  $\langle \phi_{1,2} \rangle$  become nonzero as well from electroweak symmetry breaking, the axion will acquire small components from each. Since  $S_1$  couples to the heavy singlet quarks and  $\phi_{1,2}$  to ordinary quarks, this axion is a hybrid of the two best known mechanisms [8, 9].

## 4 Particle Spectrum

In this model,  $f_N v_1$  is the mass of the heavy singlet neutrinos and  $[v_1^2 + (4/3)(u_1 + u_2 + u_3)^2]^{1/2}$  is the axion decay constant. Both are presumably of order  $10^{11 \pm 1}$  GeV to satisfy the astrophysics and cosmology bounds [10]. All superfields not belonging to the MSSM have masses of that same order, except for the following.

- (1) The very light axion of mass  $\sim 10^{-4 \pm 1}$  eV which eliminates the strong CP problem.
- (2) The corresponding physical scalar field (saxion) which has a mass  $\sim M_{SUSY}$  but whose couplings to MSSM particles are suppressed by  $v_2/v_1 \sim 10^{-8 \pm 1}$ .
- (3) The corresponding supersymmetric partner (axino) which mixes ( $\sim 10^{-9 \pm 1}$ ) with the neutralinos of the MSSM and may be the true lightest supersymmetric particle (LSP) if its mass ( $\sim 2f_1 v_2$ ) is small enough.
- (4) The two superfields orthogonal to  $(\hat{\zeta}_1 + \hat{\zeta}_2 + \hat{\zeta}_3)/\sqrt{3}$  with scalar components of order

$M_{SUSY}$  in mass and fermionic components of order  $M_{SUSY}^2/M_\chi \sim 10^{1\pm1}$  keV in mass. They decouple very effectively from all MSSM particles.

## 5 Conclusion

Merging two recent proposals, a comprehensive supersymmetric model (with just one input mass) has been presented, which results in a desirable neutrino mass matrix based on the discrete non-Abelian symmetry  $A_4$ , and removes the strong CP problem with a spontaneously broken  $U(1)_{PQ}$  symmetry, the specific application of which also conserves  $R$  parity and equates the  $\mu$  parameter with  $M_{SUSY}$ . The true lightest supersymmetric particle may be the axino of this model.

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